Contradiction between strong lensing statistics and a feedback solution to the cusp/core problem

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Standard cosmology has many successes on large scales, but faces some fundamental difficulties on small, galactic scales. One such difficulty is the cusp/core problem. High resolution observations of the rotation curves for dark matter dominated low surface brightness (LSB) galaxies imply that galactic dark matter halos have a density profile with a flat central core, whereas N-body structure formation simulations predict a divergent (cuspy) density profile at the center. It has been proposed that this problem can be resolved by stellar feedback driving turbulent gas motion that erases the initial cusp. However, strong gravitational lensing prefers a cuspy density profile for galactic halos. In this paper, we use the most recent high resolution observations of the rotation curves of LSB galaxies to fit the core size as a function of halo mass, and compare the resultant lensing probability to the observational results for the well defined combined sample of the Cosmic Lens All-Sky Survey (CLASS) and Jodrell Bank/Very Large Array Astrometric Survey (JVAS). The lensing probabilities based on such density profiles are too low to match the observed lensing in CLASS/JVAS. High baryon densities in the galaxies that dominate the lensing statistics can reconcile this discrepancy, but only if they steepen the mass profile rather than making it more shallow. The result is contradictory demands upon the effects of baryons on the central mass profiles of galaxies.

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In the standard cosmological model (known as ΛCDM), the universe is dominated by invisible components called dark energy (Λ) and cold dark matter (CDM). The ΛCDM cosmology is very successful in explaining the cosmic microwave background and the formation of large scale structure. However, there are challenges to ΛCDM on smaller scales [1]. Here we focus on the cusp/core problem [2, 3, 4] and whether proposed solutions to this problem can be consistent with the observed frequency of strong gravitational lensing.

One possible solution to the cusp/core problem is turbulence driven by stellar feedback during galaxy formation. If this process drives massive clumps of gas through the central regions of the first dark matter halos to form [5, 6], the central cusp may transform into a soft core. Once established, phase space arguments imply that the core should persist through subsequent mergers [7, 8], leading to a final halo profile with a finite core radius for all galaxies. Such a situation is consistent with essentially all kinematic observations [9, 10]. While this may

provide a satisfactory explanation for the cusp/core problem, we show here that it leads to a contradiction with the mass profiles of elliptical galaxies that dominate the statistics of strong gravitational lensing.

Gravitational lensing provides a powerful tool to detect dark matter. The lensing efficiency is very sensitive to the slope γ of the central mass density profile ($\rho \propto r^{-\gamma}$). It is well established [11, 12, 13] that when galaxies are modeled as a singular isothermal sphere (SIS: $\gamma = 2$) and galaxy clusters are modeled as a Navarro-Frenk-White (NFW: $\gamma = 1$) profile (see Table 1), the predicted strong lensing probabilities match the results from CLASS/JVAS. A steeper density slope near the center gives a more efficient lensing rate. For example, if we model galaxies with an NFW rather than SIS profile, the lensing probabilities are too low compared with observations at small image separations [12]. The presence of a central flat core ($\gamma \approx 0$) in galaxies would further limit the lensing efficiency [14]. For example, a nonsingular truncated isothermal sphere (NTIS), which is an analytical model [16] for the postcollapse equilibrium structure of virialized objects, has a soft core that matches quite well with the mass profiles of dark matter dominated LSB galaxies deduced from their observed rotation curves. The probabilities for lensing by NTIS halos are far too low compared to observations [14], however.

In order to investigate the effect of a central core on the strong lensing efficiency, we use the density profile of the halos directly constrained by observed rotation curves. These are well fit [17] by the cored isothermal sphere (CIS). The CIS halo has a finite core radius r_c within which the density is constant ($\gamma = 0$). As well as providing a good description of the data, the CIS provides

TABLE I: Halo Profiles

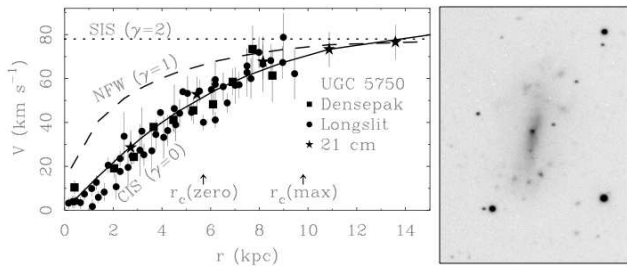| Halo	γ	$\rho(r)$
SIS	2	$\rho_0(r/r_0)^{-2}$
NFW	1	$\rho_i[(r/r_s)(1 + r/r_s)^2]^{-1}$
CIS	0	$\rho_0[1 + (r/r_c)^2]^{-1}$

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FIG. 1: The rotation curve (left) of the LSB galaxy UGC 5750 (right). Velocity data come from several independent sources and methods, including radio synthesis observations of the 21 cm spin flip transition of atomic hydrogen [18], two independent [19, 20] optical long slit observations of the n=3→2 Balmer transition (Hα), and Densepak integrated field Hα spectroscopy [21]. The various halo types are shown as lines (as marked). The parameters of NFW halos are not free, following [2, 21] from ΛCDM cosmology. The difference between this and the data is the cusp/core problem. The core radius of the CIS fit is marked by arrows for the cases of zero and maximum disk. For clarity, the full CIS halo is only shown for the case of zero disk. Attributing mass to the stellar disk detracts from the velocity that can be attributed to dark matter (albeit not much in the case of LSB galaxies), increasing r_c as shown and makes the discrepancy with the NFW prediction of ΛCDM more serious. Under no circumstances can the halos of LSB galaxies be modeled by SIS.

a reasonable proxy for the unwieldy NTIS profile.

The best objects for tracing the mass profile of the dominant dark matter component are LSB galaxies. In other galaxy types, the stellar mass can provide a non-negligible contribution to the rotation velocity at observed radii. This is not the case for LSB galaxies, whose diffuse disks remain dark matter dominated [22] down to small radii. These objects persistently suggest that dark matter halos possess approximately flat cores [23] that are best fit with CIS halos (Figure 1).

We use the most recent results [24] from a sample of LSB galaxies for which rotation curves have been derived from high-resolution optical velocity fields. For each halo, we calculate the mass M by integrating the CIS density profile to the radius r_{200}. This is the radius of a sphere within which the average mass density is 200 times the critical density of the universe, typically taken [2] as the virial radius. We compute the halo mass for two bracketing assumptions [24] about the mass of the baryonic disk: zero disk, in which the mass of stars and gas is neglected, and maximum disk, which attributes the most mass possible to the stars without exceeding the observed rotation. The primary difference between these two cases is in the core radius inferred for the halo. As more mass is attributed to the stars, less dark matter is necessary at small radii. Consequently, r_c grows with stellar mass.

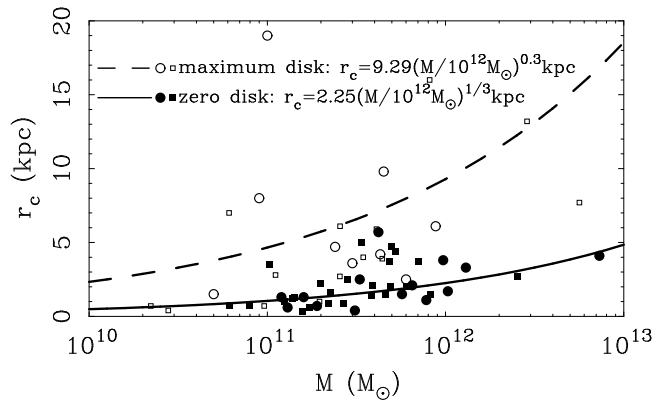There are correlations between the core radius r_c and the halo mass M in both zero disk and maximum disk

FIG. 2: The correlation between core radius r_c and halo mass M. Filled points represent the case of zero disk mass and the open points represent maximum disk. The lines are the fit to the Densepak data [24] only (circles); fits to long slit data [20] (squares) give indistinguishable resuts.

cases. Since our aim is to investigate the effect of the core radius on strong gravitational lensing efficiency, we fit the relation between r_c and M (Figure 2). As a check, we repeat the procedure with independent data [20]. The results are indistinguishable.

The gravitational lensing principle tells us that for any spherically symmetric density profile (here, a CIS halo), multiple images of a source can be produced only if the central convergence κ_c is larger than unity [25]. The central convergence is a measure of the central surface mass density of the lensing halos. It is both mass and redshift dependent. For singular density profiles such as SIS and NFW, the central value is divergent, so $\kappa_c > 1$ is always satisfied and multiple images can be produced by any mass. For density profiles with a finite soft core, however, the condition $\kappa_c > 1$ imposes a minimum mass threshold to produce multiple images. For CIS halos [14], we have $\kappa_c \propto M^{2/3}/r_c$. The larger the core radius, the larger the mass needed to ensure $\kappa_c > 1$. While both the zero and maximum disk cases give similar r_c-M relations, the more conservative case is that with the smaller core radius for a given mass; other choices would produce less lensing. We thus use the formula fit to the zero disk case: $r_c = 2.25(M/10^{12}M_\odot)^{1/3}$ kpc. Interestingly, this formula is similar to the one derived analytically in the NTIS model [14, 16].

The combined JVAS/CLASS survey forms a well-defined statistical sample containing 13 multiply imaged lens systems [26, 27] among 8,958 sources. These data provide the image separation $\Delta\theta$ for each lens system. The observational probability $P_{obs}(> \Delta\theta)$ for the CLASS/JVAS survey is shown in Figure 3.

When a remote quasar is lensed by a CIS halo, three images are produced. The image nearest the lens is very weak. It disappears entirely when the source, lens, and observer are aligned, and the Einstein ring appears. The

image separation $\Delta\theta$ is thus the separation between the outer two images. By adopting a model for the density profile of lensing halos, their comoving number density, and the geometry of the ΛCDM universe, we can predict the properties of the strong lens systems.

In order to compare with the observed lensing probabilities, we calculate $P_{\mathrm{CIS}}(> \Delta\theta)$, the lensing probability for quasars at redshift z_s lensed by foreground CIS halos with image separation larger than $\Delta\theta$. The redshift z_s of the sources (quasars) for the CLASS/JVAS sample has an approximately Gaussian distribution [11, 26] with a mean of 1.27 and a dispersion of 0.95. The lensing rate is sensitive to z_s, but the effect of the redshift distribution is negligible compared to the choice of halo profile. We thus use the mean value $z_s = 1.27$ in our calculations. For each lens system, the image separation depends on the source position. For the CIS model, however, the image separation is almost source position independent [14], so we use the diameter of the Einstein ring as the image separation for each lens system. Gravitational lensing magnifies the brightness of sources, so the number of lenses will be overrepresented [28] in any observed sample. The theoretically predicted lensing probability should therefore include a magnification bias (MB) correction to the observed probability. The MB is calculated on the basis of the total magnification of the outer two brighter images [29]. One of the most important elements in predicting lensing probability is the comoving number density of lensing galaxies. We adopt the results recently derived [30] from the Sloan Digital Sky Survey. The background cosmology is taken from the five-year Wilkinson Microwave Anisotropy Probe observations [31]. The final predicted lensing probability for CIS is plotted in Figure 3. For comparison, the lensing probability of the SIS model is shown with the same parameters and approximations as CIS. The NFW model [14] is also shown.

The predicted lensing probability for the CIS modeled galaxies is about four orders of magnitude lower than the observations of CLASS/JVAS at all image separations, and two orders of magnitude lower than the NFW model at small image separations. We have used a spherical model, as it is kown that the ellipticity does not significantly affect the total lensing efficiency[32]. Indeed, the combination of all our approximations together can shift the result by no more than one order of magnitude, as can be seen from the close match of our approximate SIS model to the data. So it is safe to conclude that halo models like CIS and NTIS with the soft central cores derived from kinematic observations can not account for the statistics of strong gravitational lensing.

It is not difficult to understand the low lensing probability of the CIS model. Recall that the central convergence depends on the mass M and the redshift z_L of the lensing halos. With the fitting formula $r_c \propto M^{1/3}$, we have $\kappa_c(M, z_L) \sim M^{1/3}F(z_L)$, where $F(z_L) = $ $\Omega(z_L)^{1/6}D_L D_{LS}/D_S$, with $\Omega(z_L) = \Omega_m(1 + z_L)^3 + \Omega_\Lambda$, D_L, D_S and D_{LS} are the angular diameter distances from the observer to the lens, to the source and from the lens to the source, respectively. For quasars at $z_S = 1.27$, $F(z_L)$ has a maximum value of 0.24 for z_L in the interval $[0, z_S]$. The condition $\kappa_c > 1$ for strong lensing implies $M > 3 \times 10^{13} M_\odot$. Since $M \sim 10^{13} M_\odot$ corresponds to the most massive galaxies in the present universe, the galaxies with lower mass provide no contribution to the total lensing probability. Furthermore, the contributions of all galaxies to the total lensing probabilities are governed by the comoving number density $n(M)$, which has a high-mass exponential cutoff [33], $n(M) \sim \exp(-M^{\beta/3})$, with $\beta = 2.67$ in our calculations. Consequently, galaxies with mass lower than $\sim 10^{13} M_\odot$ make no contribution, and high-mass galaxies meet the exponential cutoff. Some previous work [15] also used the CIS model for early-type galaxies to calculate the strong lensing probabilities, and obtained reasonable results. They adopted a typical value of the core radius of $r_c \sim 0.1$ kpc, much smaller than ours (~ 2.25 kpc), so hardly different from SIS.

The only difference between CIS and SIS is that CIS has a finite core radius. While the SIS model matches the lensing observations quite well, the low lensing probabilities of the CIS model is in serious contradiction to observations of strong gravitational lensing. Similarly, the NFW/SIS model contradicts rotation curve data. The proposed remedy [5, 6] of the cusp/core problem via feedback driven turbulence fixes this problem at the expense of creating another.

Most lensing galaxies are giant elliptical galaxies with substantial stellar masses, while we base the CIS model on observations of dark matter dominated LSB galaxies. These are very different galaxy types. Lensing is not sensitive to whether the mass doing the lensing is baryonic or dark, so the contradiction might be avoided if the total mass distribution of ellipticals — stars plus dark matter — can be modeled as SIS spheres. The challenge then becomes a self-consistent understanding of the formation of all galaxy types.

In the context of the ΛCDM structure formation paradigm, the initial condition for galaxy formation is the NFW halo. Baryonic gas dissipates and settles to the center of the gravitational potential defined by the dark matter to form the visible galaxy. As the gas collapses, the potential must adjust to the rearrangement of mass. This process, commonly referred to as adiabatic contraction[34], has the effect of steepening[35] the mass profile (increasing γ). Since the NFW halo is not adequate to explain lensing on its own[14, 36] (Figure 3), this process seems necessary to produce elliptical galaxies that behave as SIS spheres. Indeed, any transformation other than $\gamma = 1 \to 2$ is disallowed.

In LSB galaxies, we need the opposite process: something that drives γ from $1 \to 0$. This is what turbulence is proposed[5, 6] to do. The hypothesized turbulence is
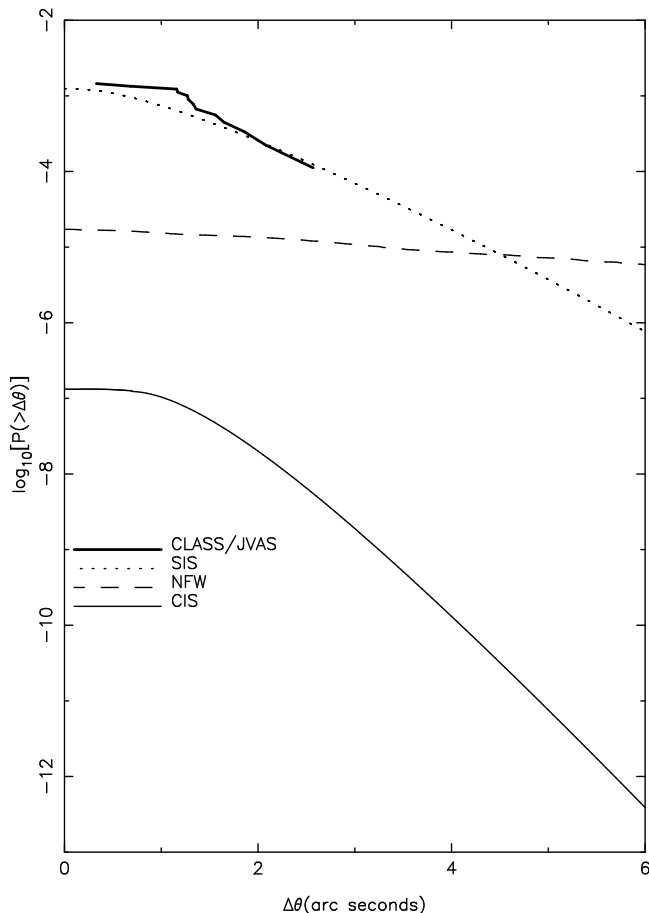
FIG. 3: The lensing probability with image separation larger than $\Delta\theta$. Our prediction for the CIS model based on the observed r_c-M relation (Figure 2) is shown as the solid line. This fails to explain the observed lensing frequency (heavy line) by four orders of magnitude. In contrast, our approximate SIS model (dotted line) provides a reasonable match to the data. A pure NFW model (dashed line) gives intermediate results.

pealing idea makes predictions that can immediately be tested. Elliptical galaxies need to steepen their mass profile ($\gamma = 1 \to 2$), implying quiescent formation in which adiabatic contraction is the dominant effect. Meanwhile, LSB galaxies need to experience vigorous bursts of early star formation that drive turbulence to erase the initial cusp ($\gamma = 1 \to 0$).

It seems quite reasonable to suppose that these very different galaxy types formed in different ways. Unfortunately, all available evidence suggests that these systems have behaved in the opposite manner form what we need. The stars in ellipticals are predominantly old[37], seeming to have formed in vigorous early bursts of star formation, perhaps associated with frequent merging. They are the result of violent, not quiescent galaxy formation. LSB galaxies, on the other hand, show no evidence [38, 39] of ever having experience intense star formation. They are the poster children for quiescent galaxy formation. This is precisely the opposite of what is needed.

We conclude that the apparent contradiction between rotation curves and strong lensing statistics pointed out here is genuine. It is difficult to simultaneously reconcile the soft cored halos favored by many kinematic observations with the singular mass profiles favored by strong lensing. In both cases, a fundamental tenet of the ΛCDM structure formation paradigm, the NFW halo, is inadequate to explain the observations. Substantial rearrangement of the initial NFW mass profile is required. Ideas hypothesized to solve one problem tend to make the other one worse.

driven by feedback from early star formation in the first halos. If this process is universal and efficient, as proposed, then we may only solve the cusp/core problem at the expense of introducing a new problem with lensing. The baryons must first collapse to the center of the halo before they can drive feedback there. So only one process can dominate: either adiabatic contraction, which increases γ, or feedback, which might reduce γ. If feedback succeeds in establishing a soft core, it should persist through subsequent mergers [7, 8]. It is difficult to see how an elliptical galaxy with an SIS mass profile could be constructed in this scenario.

If only some halos experience vigorous star formation and feedback while others form their galaxies through a gentler adiabatic contraction, we might achieve a consistent explanation with difference initial conditions leading to two very different types of galaxies. This ap-

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